

5-Dimensional Warped Cosmological Solutions With Radius Stabilization by a Bulk Scalar

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Abstract

We present the 5-dimensional cosmological solutions in the Randall-Sundrum warped compactification scenario, using the Goldberger-Wise mechanism to stabilize the size of the extra dimension. Matter on the Planck and TeV branes is treated perturbatively, to first order. The back-reaction of the scalar field on the metric is taken into account. We identify the appropriate gauge-invariant degrees of freedom, and show that the perturbations in the bulk scalar can be gauged away. We confirm previous, less exact computations of the shift in the radius of the extra dimension induced by matter. We point out that the physical mass scales on the TeV brane may have changed significantly since the electroweak epoch due to cosmological expansion, independently of the details of radius stabilization.

1. Introduction. The Randall-Sundrum (RS) idea [1] for explaining the weak-scale hierarchy problem has garnered much attention from both the phenomenology and string-theory communities, providing a link between the two which is often absent. RS is a simple and elegant way of generating the TeV scale which characterizes the standard model from a set of fundamental scales which are of order the Planck mass (M_p). All that is needed is that the distance between a hidden and a visible sector brane be approximately $b = 35/M_p$ in a compact extra dimension, $y \in [0, 1]$. The warping of space in this extra dimension, by a factor e^{-kby} , translates the moderately large interbrane separation into the large hierarchy needed to explain the ratio TeV/M_p .

However the RS idea as originally proposed was incomplete due to the lack of any mechanism for stabilizing the brane separation, b . This was a modulus, corresponding to a massless particle, the radion, which would be ruled out because of its modification of gravity: the attractive force mediated by the radion would effectively increase Newton's constant at large distance scales. An attractive model for giving the radion a potential energy was proposed by Goldberger and Wise (GW) [2]; they introduced a bulk scalar field with different VEV's, v_0 and v_1 , on the two branes. If the mass m of the scalar is small compared to the scale k which appears in the warp factor e^{-kby} , then it is possible to obtain the desired interbrane separation. One finds the relation $e^{-kb} \cong (v_1/v_0)^{4k^2/m^2}$.

An important benefit of stabilizing the radion is that cosmology is governed by the usual Friedmann equations, up to small corrections of order $\rho/(\text{TeV})^4$ [3]. Even with stabilization, there may be a problem with reaching a false minimum of the GW radion potential [4], but without stabilization, there is a worse problem: an unnatural tuning of the energy densities on the two branes is required for getting solutions where the extra dimension is static [5, 6], a result which can be derived using the (5,5) component of the Einstein equation $G_{mn} = \kappa^2 T_{mn}$. However when there is a nontrivial potential for the radius, $V(b)$, the (5,5) equation serves only to determine the shift δb in the radius due to the expansion, and there is no longer any constraint on the matter on the branes. Although this point is now well appreciated [7]-[10], it has not previously been explicitly demonstrated by solving the full 5-dimensional field equations using a concrete stabilization mechanism. Indeed, it has been claimed recently that such solutions are not possible with an arbitrary equation of state for the matter on the branes [12]-[13], and also that the rate of expansion does not reproduce normal cosmology on the negative tension brane despite stabilization [14]. Our purpose is to present the complete solutions, to leading order in an expansion in the energy densities on the branes, thus refuting these claims.

2. Preliminaries. The action for 5-D gravity coupled to the stabilizing scalar field Φ and matter on the branes (located at $y = 0$ and $y = 1$, respectively) is

$$\begin{aligned}
S = & \int d^5x \sqrt{g} \left(-\frac{1}{2\kappa^2} R - \Lambda + \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) \right) \\
& + \int d^4x \sqrt{g} (\mathcal{L}_{m,0} - V_0(\Phi))|_{y=0} + \int d^4x \sqrt{g} (\mathcal{L}_{m,1} - V_1(\Phi))|_{y=1},
\end{aligned} \tag{1}$$

where κ^2 is related to the 5-D Planck scale M by $\kappa^2 = 1/(M^3)$. The negative bulk cosmological constant needed for the RS solution is parametrized as $\Lambda = -6k^2/\kappa^2$ and the scalar field

potential is that of a free field, $V(\Phi) = \frac{1}{2}m^2\Phi^2$. The brane potentials V_0 and V_1 can have any form that will insure nontrivial VEV's for the scalar field at the branes, for example $V_i(\Phi) = \lambda_i(\Phi^2 - v_i^2)^2$ [2]. In ref. [4] we pointed out that the choice $V_i(\Phi) = m_i(\Phi - v_i)^2$ is advantageous from the point of view of analytic calculability (see also [15]).

We will take the metric to have the form

$$\begin{aligned} ds^2 &= n^2(t, y)dt^2 - a^2(t, y) \sum_i dx_i^2 - b^2(t, y)dy^2 \\ &= e^{-2N(t, y)}dt^2 - a_0(t)^2 e^{-2A(t, y)} \sum_i dx_i^2 - b(t, y)^2 dy^2, \end{aligned} \quad (2)$$

where a perturbative expansion in the energy densities of the branes will be made around the static solution:

$$\begin{aligned} N(t, y) &= A_0(y) + \delta N(t, y); & A(t, y) &= A_0(y) + \delta A(t, y) \\ b(t, y) &= b_0 + \delta b(t, y); & \Phi(t, y) &= \Phi_0(y) + \delta \Phi(t, y). \end{aligned} \quad (3)$$

The perturbations are taken to be linear in the energy densities ρ_* and ρ of matter on the Planck and TeV branes, located at $y = 0$ and $y = 1$, respectively.

This ansatz is to be substituted into the Einstein equations, $G_{mn} = \kappa^2 T_{mn}$, and the scalar field equation

$$\partial_t \left(\frac{1}{n} b a^3 \dot{\Phi} \right) - \partial_y \left(\frac{1}{b} a^3 n \Phi' \right) + b a^3 n [V' + V_0' \delta(by) + V_1' \delta(b(y-1))] = 0. \quad (4)$$

Here and in the following, primes on functions of y denote $\frac{\partial}{\partial y}$, while primes on potentials of Φ will mean $\frac{\partial}{\partial \Phi}$. The nonvanishing components of the Einstein tensor are

$$\begin{aligned} G_{00} &= 3 \left[\left(\frac{\dot{a}}{a} \right)^2 + \frac{\dot{a}}{a} \frac{\dot{b}}{b} - \frac{n^2}{b^2} \left(\frac{a''}{a} + \left(\frac{a'}{a} \right)^2 - \frac{a'b'}{ab} \right) \right] \\ G_{ii} &= \frac{a^2}{b^2} \left[\left(\frac{a'}{a} \right)^2 + 2 \frac{a'}{a} \frac{n'}{n} - \frac{b'}{b} \frac{n'}{n} - 2 \frac{b'}{b} \frac{a'}{a} + 2 \frac{a''}{a} + \frac{n''}{n} \right] \\ &\quad + \frac{a^2}{n^2} \left[- \left(\frac{\dot{a}}{a} \right)^2 + 2 \frac{\dot{a}}{a} \frac{\dot{n}}{n} - 2 \frac{\ddot{a}}{a} + \frac{\dot{b}}{b} \left(-2 \frac{\dot{a}}{a} + \frac{\dot{n}}{n} \right) - \frac{\ddot{b}}{b} \right] \\ G_{05} &= 3 \left[\frac{n'}{n} \frac{\dot{a}}{a} + \frac{a'}{a} \frac{\dot{b}}{b} - \frac{\dot{a}'}{a} \right] \\ G_{55} &= 3 \left[\frac{a'}{a} \left(\frac{a'}{a} + \frac{n'}{n} \right) - \frac{b^2}{n^2} \left(\frac{\dot{a}}{a} \left(\frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) + \frac{\ddot{a}}{a} \right) \right] \end{aligned} \quad (5)$$

and the stress energy tensor is $T_{mn} = g_{mn}(V(\Phi) + \Lambda) + \partial_m \Phi \partial_n \Phi - \frac{1}{2} \partial^l \Phi \partial_l \Phi g_{mn}$ in the bulk. On the branes, T_m^n is given by

$$\begin{aligned} T_m^n &= \delta(by) \text{diag}(V_0 + \rho_*, V_0 - p_*, V_0 - p_*, V_0 - p_*, 0) \\ &\quad + \delta(b(y-1)) \text{diag}(V_1 + \rho, V_1 - p, V_1 - p, V_1 - p, 0) \end{aligned} \quad (6)$$

At zeroth order in the perturbations, the equations of motion can be written as

$$\begin{aligned} A_0'^2 &= \frac{\kappa^2}{12} (\Phi_0'^2 - m^2 b_0^2 \Phi_0^2) + k^2 b_0^2; & A_0'' &= \frac{1}{3} \kappa^2 \Phi_0'^2 \\ \Phi_0'' &= 4A_0' \Phi_0' + m^2 b_0^2 \Phi_0, \end{aligned} \quad (7)$$

and the solutions are approximately

$$\Phi_0(y) \cong v_0 e^{-\epsilon k b_0 y}; \quad A_0(y) \cong k b_0 y + \frac{\kappa^2}{12} v_0^2 (e^{-2\epsilon k b_0 y} - 1) \quad (8)$$

where we have normalized $A_0(0) = 0$, and introduced

$$\epsilon = \sqrt{4 + \frac{m^2}{k^2}} - 2 \cong \frac{m^2}{4k^2}. \quad (9)$$

The above approximation is good in the limit $\epsilon \ll 1$, which is the same regime in which the Goldberger-Wise mechanism naturally gives a large hierarchy without fine-tuning the scalar field VEV's on the branes: $e^{-k b_0} = (v_1/v_0)^{1/\epsilon}$. For small ϵ , the GW solution coincides with an exact solution of the coupled equations that was presented in ref. [15].

3. Perturbation Equations. We can now write the equations for the perturbations of the metric, δA , δN , δb , and the scalar field, $\delta \Phi$. The equations take a simpler form when expressed in terms of the following combinations:

$$\Psi = \delta A' - A_0' \frac{\delta b}{b_0} - \frac{\kappa^2}{3} \Phi_0' \delta \Phi; \quad \Upsilon = \delta N' - \delta A' \quad (10)$$

Further simplification comes from realizing that the perturbations will have the form, for example, $\Psi = \rho_*(t) g_0(y) + \rho(t) g_1(y)$, so that their time derivatives are proportional to $\dot{\rho}$ and $\dot{\rho}_*$. Below we will confirm that $\dot{\rho} = -3H(\rho + p)$, where $H \sim \sqrt{\rho}$, $\sqrt{\rho_*}$ is the Hubble parameter. Therefore time derivatives of the perturbations are higher order in ρ and ρ_* than are y derivatives, and can be neglected at leading order (except in the (05) Einstein equation, where $\rho^{3/2}$ is the leading order). Using this approximation, we can write the combinations (00), (00)–(ii), (05) and (55) of the Einstein equations as

$$4A_0' \Psi - \Psi' = \left(\frac{\dot{a}_0}{a_0} \right)^2 b_0^2 e^{2A_0} \quad (11)$$

$$-4A_0' \Upsilon + \Upsilon' = 2 \left(\left(\frac{\dot{a}_0}{a_0} \right)^2 - \frac{\ddot{a}_0}{a_0} \right) b_0^2 e^{2A_0} \quad (12)$$

$$-\frac{\dot{a}_0}{a_0} \Upsilon + \dot{\Psi} = 0 \quad (13)$$

$$A_0' (4\Psi + \Upsilon) + \frac{\kappa^2}{3} \left(\Phi_0'' \delta \Phi - \Phi_0' \delta \Phi' + \Phi_0'^2 \frac{\delta b}{b_0} \right) = \left(\left(\frac{\dot{a}_0}{a_0} \right)^2 + \frac{\ddot{a}_0}{a_0} \right) b_0^2 e^{2A_0} \quad (14)$$

In addition, there is the scalar field equation,

$$\delta\Phi'' = (4\Psi + \Upsilon)\Phi'_0 + \left(\frac{4\kappa^2}{3}\Phi_0'^2 + b_0^2 V''(\Phi_0)\right) \delta\Phi + 4A_0' \delta\Phi' + \left(2b_0^2 V'(\Phi_0) + 4A_0' \Phi_0'\right) \frac{\delta b}{b_0} + \Phi_0' \frac{\delta b'}{b_0} \quad (15)$$

Assuming Z_2 symmetry (all functions symmetric under $y \rightarrow -y$), the boundary conditions implied by the delta function sources at the branes are

$$\Psi(t, 0) = +\frac{\kappa^2}{6}b_0\rho_*(t); \quad \Psi(t, 1) = -\frac{\kappa^2}{6}b_0\rho(t) \quad (16)$$

$$\Upsilon(t, 0) = -\frac{\kappa^2}{2}b_0(\rho_* + p_*)(t); \quad \Upsilon(t, 1) = +\frac{\kappa^2}{2}b_0(\rho + p)(t) \quad (17)$$

$$\delta\Phi'(t, y_n) = \frac{\delta b(t, y_n)}{b_0}\Phi_0'(t, y_n) + (-1)^n \left(\frac{b_0}{2}\right) V_n''(\Phi_0(t, y_n)) \delta\Phi(t, y_n), \quad (18)$$

where in (18) $n = 0, 1$, $y_0 = 0$ and $y_1 = 1$.

4. Solutions. Naively, it would appear that we have five equations for four unknown perturbations, but of course since gravity is a gauge theory, this is not the case. First, we have the relation $\frac{\partial}{\partial t}[\text{Eq. 11}] + \frac{\dot{a}_0}{a_0}[\text{Eq. 12}] = [\text{Eq. 13}]$. Furthermore, the (55) Einstein equation and the scalar equation can be shown to be equivalent, using (00),(ii) and the zeroth order relations (7): $[\text{Eq. (14)}]' - 4A_0' \times [\text{Eq. (14)}] = \Phi_0' \times [\text{Eq. (15)}]$. So our system is actually underdetermined because of unfixed gauge degrees of freedom. To see this more directly, consider an infinitesimal diffeomorphism which leaves the coordinate positions of the branes unchanged: $y = \bar{y} + f(\bar{y})$, where $f(0) = f(1) = 0$. The metric and scalar perturbations transform as

$$\begin{aligned} \delta A &\rightarrow \delta A + A_0' f; & \delta N &\rightarrow \delta N + A_0' f \\ \delta b &\rightarrow \delta b + b_0 f'; & \delta \Phi &\rightarrow \delta \Phi + \Phi_0' f \end{aligned} \quad (19)$$

If desired, one can form the gauge invariant combinations

$$\delta A' - A_0' \frac{\delta b}{b_0} - \frac{\kappa^2}{3} \Phi_0' \delta \Phi; \quad \delta N' - \delta A'; \quad \Phi_0'' \delta \Phi - \Phi_0' \delta \Phi' + \Phi_0'^2 \frac{\delta b}{b_0} \quad (20)$$

the first two are precisely our variables Ψ and Υ and the last one appears in (55) equation. In terms of these gauge invariant variables, the system of equations closes.

It is now easy to verify the following solution from the (00) and (00)-(ii) equations, *i.e.*, eqs. (11-12). Denoting the warp factor $\Omega = e^{-A_0(1)}$, we find

$$\Psi = \frac{\kappa^2 b_0}{6(1 - \Omega^2)} e^{4A_0(y)} \left(F(y)(\Omega^4 \rho + \rho_*) - (\Omega^4 \rho + \Omega^2 \rho_*) \right) \quad (21)$$

$$\Upsilon = \frac{\kappa^2 b_0}{2(1 - \Omega^2)} e^{4A_0(y)} \left(-F(y)(\Omega^4(\rho + p) + \rho_* + p_*) + (\Omega^4(\rho + p) + \Omega^2(\rho_* + p_*)) \right) \quad (22)$$

where

$$F(y) = 1 - (1 - \Omega^2) \frac{\int_0^y e^{-2A_0} dy}{\int_0^1 e^{-2A_0} dy} \cong e^{-2kb_0 y} \quad (23)$$

and the Friedmann equations are

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{8\pi G}{3} (\rho_* + \Omega^4 \rho) \quad (24)$$

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 - \frac{\ddot{a}_0}{a_0} = 4\pi G (\rho_* + p_* + \Omega^4(\rho + p)) \quad (25)$$

$$8\pi G = \kappa^2 \left(2b_0 \int_0^1 e^{-2A_0} dy\right)^{-1} \cong \kappa^2 k (1 - \Omega^2)^{-1}. \quad (26)$$

The approximations in eqs. (23) and (26) hold when the back reaction of the scalar field on the metric can be neglected.

In the Friedmann equations (24-25), we note that ρ is the bare value of the energy density on the TeV brane, naturally of order M_p^4 , while $\Omega^4 \rho$ is the physically observable value, of order $(\text{TeV})^4$. Since ρ_* has no such suppression, it seems highly unlikely that ρ_* should be nonzero today; otherwise it would tend to vastly dominate the present expansion of the universe. We also point out that these equations are consistent only if energy is separately conserved on each brane: $\dot{\rho} + 3H(\rho + p) = 0$ and $\dot{\rho}_* + 3H(\rho_* + p_*) = 0$. This can be derived directly by considering the (05) Einstein equation, evaluated at either of the branes. The equations of state on the two branes are completely independent; there is no relation between p/ρ and p_*/ρ_* .

5. Stiff potential limit. The above solutions are quite general, but they are not complete because we have not yet solved for the scalar field perturbation, $\delta\Phi$. This would generically be intractable, but there is a special case in which things simplify, namely, when the brane potentials $V_i(\Phi)$ become stiff. In this case, the boundary condition for the scalar fluctuation becomes $\delta\Phi = 0$ at either brane. There is no information about the derivative $\delta\Phi'$ in this case; although $\delta\Phi \rightarrow 0$, at the same $V''(\Phi) \rightarrow \infty$ in such a way that the product $\delta\Phi V''(\Phi)$ remains finite, and eq. (18) is automatically satisfied.

Notice that the shift in $\delta\Phi$, eq. (19), respects the boundary conditions on $\delta\Phi$. Moreover, Φ'_0 is always nonzero for our solution. It is therefore always possible, given some solution $\delta\Phi$ which vanishes at the branes, to choose an f such that $\delta\Phi$ becomes zero. This is a convenient choice of gauge because it simplifies the equations of motion, and we will make it for the remainder of this letter.¹ Thus far we have satisfied the (00), (ii) and (05) Einstein equations. As noted above, eqs. (14) and (15) are equivalent, so either one just determines the shift in the radius. Using the former, and defining

$$G(y) = \left[\frac{1}{2} e^{2A_0(y)} + A'_0 e^{4A_0(y)} \int_0^y e^{-2A_0} dy \right] / \int_0^1 e^{-2A_0} dy \cong \frac{kb_0 e^{4kb_0 y}}{1 - \Omega^2}, \quad (27)$$

¹The above argument is strictly true only for diffeomorphisms which are constant in time, while for our problem we need $f(t, y) \sim \rho(t), \rho_*(t)$. However, the time variation of such an f is of higher order in ρ and ρ_* , so we can neglect it to leading order in the perturbations.

we find that

$$\begin{aligned}\frac{\delta b}{b_0} &= \frac{b_0}{2\Phi_0'^2} \left[\Omega^4(\rho - 3p)G + (\rho_* - 3p_*)(G - A'_0 e^{4A_0}) \right] \\ &\cong \frac{kb_0^2 e^{4kb_0 y}}{2\Phi_0'^2(1 - \Omega^2)} \left[\Omega^4(\rho - 3p) + \Omega^2(\rho_* - 3p_*) \right];\end{aligned}\quad (28)$$

the last expression is found by approximating $A_0 = kb_0 y$ everywhere, which means neglecting the back reaction. Using the zeroth order solution (8) for Φ_0 , and integrating over y , we can obtain the shift in the size of the extra dimension,

$$\int_0^1 \delta b dy \cong \frac{[\Omega^4(\rho - 3p) + \Omega^2(\rho_* - 3p_*)]}{8(\epsilon k v_0)^2 \Omega^{4+2\epsilon}} \quad (29)$$

We can compare this to the result of ref. [16] by using their result for the radion mass, $m_r^2 \cong (4/3)\kappa^2(\epsilon v_0 k)^2 \Omega^{2+2\epsilon}$, and the relation $k\kappa^2 \cong 1/M_p^2$. Then

$$\frac{\int_0^1 \delta b dy}{b_0} \cong \frac{[\Omega^4(\rho - 3p) + \Omega^2(\rho_* - 3p_*)]}{6kb_0 m_r^2 M_p^2 \Omega^2} \quad (30)$$

which agrees with ref. [16], except for small corrections of order $(1 + \Omega^2)$.² As is well known, the shift in the radion vanishes when the universe is radiation dominated, because the radion couples to the trace of the stress energy tensor, which vanishes if the matter is conformally invariant.

6. Implications. Above we focused on the shift in the size of the extra dimension due to cosmological expansion, but the more experimentally relevant quantity is the shift in the lapse function, $n(t, 1)$, evaluated on the TeV brane. As emphasized in ref. [11], the change in $n(t, 1)$ between the present and the past determines how much physical energy scales on our brane, like the weak scale, M_W , have evolved. The time dependence of M_W is given by $M_W(t)/M_W(t_0) = e^{-\delta N(t, 1) + \delta N(t_0, 1) + \delta N(t, 0) - \delta N(t_0, 0)}$. In terms of the variables of the previous section, $\delta N' = \Psi + \Upsilon + A'_0 \delta b/b_0$. We find that

$$\int_0^1 \delta N'(t, y) dy \cong \frac{\kappa^2 b_0}{24kb_0} \left((2\rho + 3p) - e^{2kb} (2\rho_* + 3p_*) \right) + \int_0^1 A'_0 \frac{\delta b}{b_0} dy \quad (31)$$

Interestingly, the new non- δb contribution is present even during radiation domination, is parametrically smaller than the radion part only in the matter dominated era, and then only if the back reaction is small ($\epsilon \ll 1$). If $M_p \Omega \sim 1$ TeV, the shift in the energy scale since nucleosynthesis is negligible, and this is the only cosmological constraint on δN . However, near the weak scale, the effect could be more interesting. To first order in the physical energy density $\rho_p = \Omega^4 \rho$, at early times,

$$M_W(t) \cong M_W(t_0) \left(1 - \frac{\rho_p(t)}{8\Omega^4 M_p^2 k^2} \right), \quad (32)$$

²Our correction factor is $(1 - \Omega^{4+2\epsilon})/(1 - \Omega^2)$, while that of ref. [16] is $(1 - \Omega^2)$.

assuming that $\rho_* = p_* = 0$. It is conceivable that $k \sim M_p/30$ —in fact, the RS model requires $k < M_P$ for consistency, so that higher dimension operators in the gravitational part of the action do not become important. With these parameters and assuming $g_* \sim 100$ relativistic degrees of freedom and $\Omega M_p = 1$ TeV, the correction to M_W becomes of order unity at a temperature of 130 GeV. Thus the temporal variation in fundamental mass scales might have some relevance for the electroweak phase transition and baryogenesis.

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